Engineering Notes

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Efficient Method for Calculating the Axial Velocities Induced Along Rotating Blades by Trailing Helical Vortices

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Introduction

ROTATING blades are well known elements which appear in many engineering applications such as aircraft propellers marine propellers helicopter rotors wind turbine rotors etc In order to calculate the aerodynamic behavior of rotating blades it is common to replace them by lifting lines[1] having varying circulation along their span As a result of the variation of the circulation along the blade vortex filaments are trailed behind it Because of the axial velocity of the fluid and the rotating motion of the blade, helical vortex lines are obtained which create the wake of the propeller or rotor This model appears in the case of aircraft propellers [1 2] marine propellers [3 4] helicopter rotors [5] and wind turbine rotors [6] The indicated references are only representative examples of the vast use of this model over the years

In most of the cases one is only interested in the axial component of the induced velocity (parallel to the axis of rotation) Therefore the following Note will concentrate on the calculation of this component Unfortunately, as is well known to those who have tried to calculate the induced velocity, the calculations include very tedious integrations These integrations cannot be done analytically and a numerical approach is necessary The purpose of the present Note is to present an efficient method of calculating the induced velocities

Theoretical Derivation and Results

The problem which is considered here is that of b equally spaced blades which are rotating with a constant velocity Ω In Fig 1 the case of three blades is presented According to the lifting line theory, the blades are replaced by vortex lines which lie along the forward quarter chord of the blades Generally the magnitude of the circulation varies along the blade and so vortex filaments are trailed behind the blades As mentioned above, these vortex filaments have a helical shape In Fig 1 the b vortex filaments of intensity Γ_f which leave the blades at the cross section $R\ \eta$ are shown where R is the radius of the disc. Each element of the vortex filament moves in an axial velocity v relative to the disc plane The velocity which is induced at the cross section $R\ x$ by the above mentioned b helical vortex filaments is denoted $v_{f(x)}$ (v_f is positive in the $-z_{\mathrm{HUB}}$ direction) The expression for v_f can be found in many of the above mentioned References See, for example Ref 4, App 2 or Ref 5, Eq (6 7) It can be written as follows:

$$\bar{v}_{f(x)} = \int_0^\infty IV_f d\nu \tag{1}$$

where

$$\bar{v}_f = \frac{4\pi R}{\Gamma_f} v_f \tag{2}$$

$$IV_f = \sum_{n=1}^{b} \frac{\eta^2 - \eta x \cos(\gamma_n - \nu)}{[\bar{v}'^2\nu^2 + \eta^2 + x^2 - 2\eta x \cos(\gamma_n - \nu)]^{3/2}} \tag{3}$$

$$\bar{v} = \frac{v}{\Omega R} \tag{4}$$

$$\gamma_n = 2\pi(n-1)/b \tag{5}$$

$\bar{v}_f$ is a nondimensional induced velocity IV_f is the integrand of Eq (1) which is a function of b ν v x and η It turns out that IV_f shows different behavior for the cases where $x>\eta$ and $x<\eta$ as is shown in Figs 2 and 3 These figures describe the case of three blades, while $\bar{v} = 0\ 05$ $\eta = 0\ 6$ and $x = 0\ 3$ and $0\ 9$ in Figs 2 and 3 respectively

It is also clear from Figs 2 and 3 that the integration should include many revolutions in order to achieve good accuracy

Examination of Eq. (3) indicates that IV_f is usually bounded by the functions IV_f' and IV_f'' which are defined as follows (see Figs 2 and 3):

$$IV_f' = \sum_{n=1}^{b} \frac{\eta^2 - \eta x \cos\gamma_n}{[\bar{v}'^2\nu^2 + \eta^2 + x^2 - 2\eta x \cos\gamma_n]^{3/2}} \tag{6}$$

$$IV_f'' = \sum_{n=1}^{b} \frac{\eta^2 - \eta x \cos(\gamma_n + \pi/b)}{[\bar{v}'^2\nu^2 + \eta^2 + x^2 - 2\eta x \cos(\gamma_n + \pi/b)]^{3/2}} \tag{7}$$

Examination of Eqs (3) (6), and (7) and Figs 2 and 3 indicates that for sufficient high values of ν it is possible to assume that:

$$IV_f \cong ½ (IV_f' + IV_f'') \tag{8}$$

Therefore, instead of Eq (1) the following approximation may be used:

$$\bar{v}_{f(x)} \cong \int_0^{\nu_m} IV_f d\nu + ½ \left(\int_{\nu_m}^\infty IV_f' d\nu + \int_{\nu_m}^\infty IV_f'' d\nu \right) \tag{9}$$

The main advantage of Eq (9) is that the two integrals in the parentheses have a closed analytic solution as follows:

$$\int_{\nu_m}^\infty IV_f' d\nu = \sum_{n=1}^{b} \frac{\eta^2 - \eta x \cos\gamma_n}{\bar{v}'(\eta^2 + x^2 - 2\eta x \cos\gamma_n)} \times \left[1 - \nu_m \Big/ \sqrt{\nu_m^2 + \frac{\eta^2 + x^2 - 2\eta x \cos\gamma_n}{\bar{v}'^2}} \right] \tag{10}$$

$$\int_{\nu_m}^\infty IV_f'' d\nu = \sum_{n=1}^{b} \frac{\eta^2 - \eta x \cos(\gamma_n + \pi/b)}{\bar{v}'(\eta^2 + x^2 - 2\eta x \cos(\gamma_n + \pi/b))} \times \left[1 - \nu_m \Big/ \sqrt{\nu_m^2 + \frac{\eta^2 + x^2 - 2\eta x \cos(\gamma_n + \pi/b)}{\bar{v}'^2}} \right] \tag{11}$$

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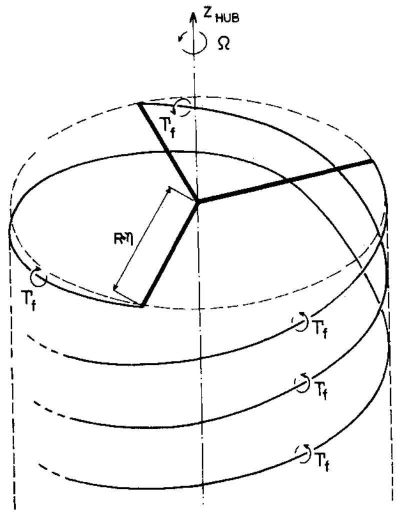

Fig 1 The geometry of the vortex lines which depart from the cross section η of the blades of a three bladed rotor

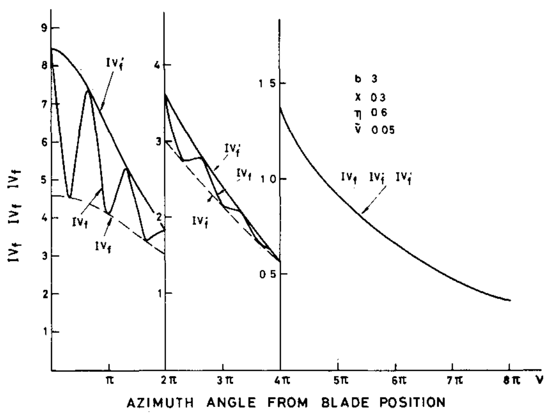

Fig 2 The behavior of IV_f IV_f' and IV_f'' as a function of ν

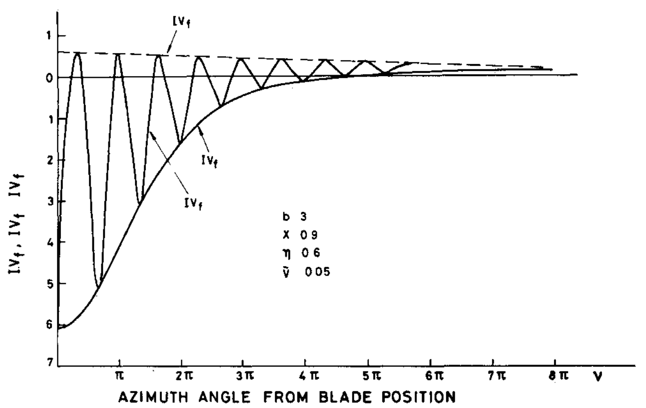

Fig 3 The behavior of IV_f IV_f' and IV_f'' as a function of ν

It is clear that in general the approximation of Eq (9) improves as ν_m is increased It is also clear that the accuracy of the approximation will also be a function of $\bar{v}$ b η and x

In Figs 4 and 5 the advantages of the approximation are presented Results of using the approximations are compared to pure integration from $\nu=0$ to $\nu=\nu_m$ In Fig 4 the case of $x<\eta$ is treated First of all as mentioned above it is shown that if pure integration according to Eq (1) is used a few revolutions of the wake should be followed in order to obtain a satisfactory convergence As shown in the figure at $\nu_m=9\pi$ there is still a deviation of 7% from the exact value The

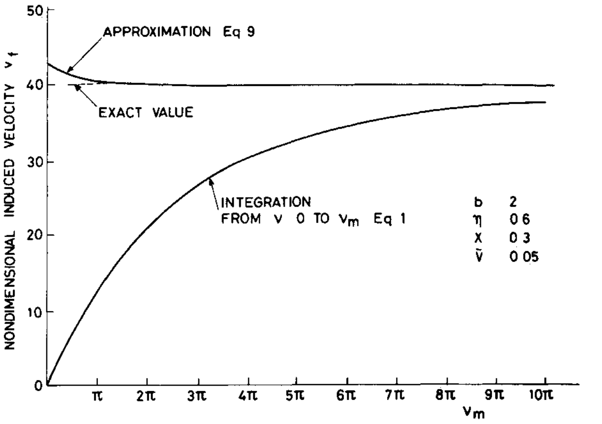

Fig 4 Comparison between the accurate and approximate integrations

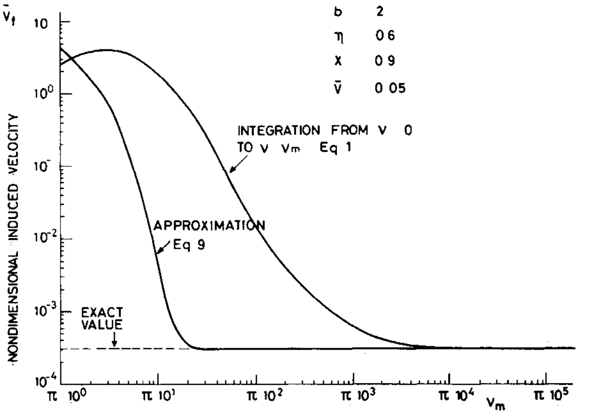

Fig 5 Comparison between the accurate and approximate integrations

convergence toward the exact value is relatively slow On the other hand use of the approximation according to Eq (9) practically yields the exact value at $\nu_m = 1\ 5\pi$ The advantages of using the approximate analysis are evident These advantages are even more dramatic in the case where $x>\eta$ as presented by Fig 5 In this case the induced velocity is very small At small values of ν_m both methods give values of induced velocity which are four order of magnitudes higher than the exact values The pure integration converges only at values of ν_m which are larger than $10^4\pi$, while the approximate method converges at $\nu_m=25\pi$ This means that use of the approximate method reduces the region of integration by almost three orders of magnitude.

The same trend of large savings by using the approximate method has been found in the case of rotors containing other numbers of blades

In Ref 7 the method which has been presented in this Note is used in order to investigate the behavior of the axial velocities induced along rotating blades by trailing helical vortices The influence of different parameters is investigated and the possibility of unified representation is discussed. It is shown that the results which are obtained for certain values of η can be applied using certain transformations for other values of η It is also shown in Ref 7 that the results of the present Note converge to the well known results for infinite number of blades

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Divergence Boundary Prediction from Random Responses: NAL's Method

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Introduction

IN Refs 1 and 2 we presented the National Aerospace Laboratory (NAL) method which can predict both flutter and divergence boundaries in an unified manner from tur bulence excited responses at subcritical speeds The estimation method[3] consists of the following procedures A time series of the response of the aeroelastic system to the Gaussian random noise input is represented by an autoregressive moving average (Æ MA) process Instead of the Æ MA process an autoregressive (Æ) process may also be used The order and coefficients of the process are estimated by Akaike's AIC minimum procedure [4] Stability of the system is evaluated with the aid of Jury's stability criteria[5] in which the estimated Æ coefficients are used The flutter or divergence boundary is determined by a least squares fit of a straight line or a parabolic curve to each set of stability parameters plotted against the dynamic pressure In addition to the boundary the modal frequencies and damping ratios are calculated from the Æ coefficients

As an example of possible application the method was applied to response signals of a cantilever wing model measured in a low supersonic, subcritical flutter test The comparison between the acutal and estimated flutter boun daries showed that an accurate prediction could be made using the data obtained in a narrow dynamic pressure range sufficiently below the boundary It was also demonstrated that no prediction could be made with the aid of the con ventional method based on the damping ratio if one resorted to only a set of the estimated damping ratios obtained in the same narrow range of the dynamic pressure [1 2 6]

The objective of this brief Note is to show that the NAL estimation method is also applicable to subcritical divergence testing In most cases divergence is the aeroelastic mode of instability for a forward swept wing Recent progress in advanced composite materials makes it now possible for aeronautical engineers to design aircrafts having wings with no serious weight penalty However little experimental in formation on divergence has been accumulated to date Therefore it is important to develop an accurate rapid and low cost technique for predicting the divergence boundary and characteristics without destruction of expensive tailored composite models during testing

Ricketts and Doggett[7] performed divergence tests on flat aluminum plate wings at transonic speeds to apply four static and two dynamic techniques and evaluate their accuracy in prediction the divergence dynamic pressure The static methods required a process of stepping the model through an angle of attack range and acquiring mean strains at each angle while keeping the flow constant It was necessary to repeat this process for several dynamic pressures in order to predict the divergence boundary In the dynamic methods a spectrum analyzer was used to determine the modal frequency and peak amplitude from the turbulence excited responses of the models fixed at zero angle of attack According to Ricketts and Dogget's evaluation the former are more reliable and accurate than the latter This is because all the static techniques are more elaborate They are however more time consuming

The NAL method is classified as a dynamic approach in which the estimation of aeroelastic characteristics of wings is based on an advanced time series analysis theory

Model and Test Procedure

Although three wing models were tested we will focus here on one of them because the results for all three models aie similar The model was constructed of an aluminum alloy flat plate of 4 mm thickness The plate was double wedged at the leading and trailing edges Figure 1 shows the wing planform strain gauge position first three natural frequencies and nodal lines of the second and third modes The experiment was conducted in the blowdown type supersonic wind tunnel of the 1×1 m test section at the NAL The strain was measured at fourteen dynamic pressures from $Q = 0\ 716$ to $1\ 015$ kg/cm^2 with the Mach number fixed at 2 5 The signal was recorded on an FM magnetic recorder After completion of the 14 measurements at constant flow conditions the dynamic pressure was increased until divergence actually occurred in order to determine the divergence dynamic pressure $Q_D = 1\ 085$ kg/cm^2 The dynamic pressure was increased at a rate of 0 026 kg/cm^2/s

Data Analysis and Results

Data analysis of the subcritical response signals for divergence is essentially the same as that for the flutter

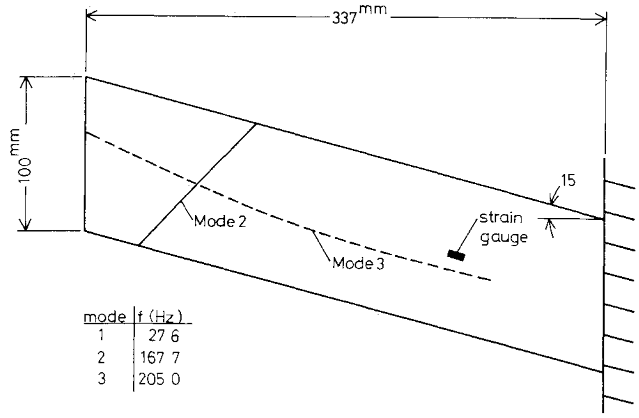

Fig 1 Forward swept wing configuration strain gauge location natural frequencies and nodal lines

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